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BIOSTATISTICS – RULES OF PROBABILITY

PG Sem 2 CC-6 Unit V

Introduction:

Biostatistics is the application of statistical principles to questions and problems in medicine, public health or biology. One can imagine that it might be of interest to characterize a given population (e.g., adults in Boston or all children in the United States) with respect to the proportion of subjects who are overweight or the proportion who have asthma, and it would also be important to estimate the magnitude of these problems over time or perhaps in different locations. In other circumstances in would be important to make comparisons among groups of subjects in order to determine whether certain behaviors (e.g., smoking, exercise, etc.) are associated with a greater risk of certain health outcomes. It would, of course, be impossible to answer all such questions by collecting information (data) from all subjects in the populations of interest. A more realistic approach is to study samples or subsets of a population. The discipline of biostatistics provides tools and techniques for collecting data and then summarizing, analyzing, and interpreting it. If the samples one takes are representative of the population of interest, they will provide good estimates regarding the population overall. Consequently, in biostatistics one analyzes samples in order to make inferences about the population. This module introduces fundamental concepts and definitions for biostatistics.

What is Probability ?

Probability means possibility. It is a branch of mathematics that deals with the occurrence of a random event. The value is expressed between zero and one. Probability has been introduced in Maths to predict how likely events are to happen.

The meaning of probability is basically the extent to which something is likely to happen. This is the basic probability theory which is also used in the probability distribution, where you will learn the possibility of outcomes for a random experiment. To find the probability of a single event to occur, first we should know the total number of possible outcomes.

Definitions and Notations

Two events are mutually exclusive or disjoint if they cannot occur at the same time.

The probability that Event A occurs, given that Event B has occurred, is called a conditional probability. The conditional probability of Event A, given Event B, is denoted by the symbol $P(A|B)$.

The complement of an event is the event not occurring. The probability that Event A will not occur is denoted by $P(A')$.

The probability that Events A and B both occur is the probability of the intersection of A and B. The probability of the intersection of Events A and B is denoted by $P(A \cap B)$. If Events A and B are mutually exclusive, $P(A \cap B) = 0$.

The probability that Events A or B occur is the probability of the union of A and B. The probability of the union of Events A and B is denoted by $P(A \cup B)$.

If the occurrence of Event A changes the probability of Event B, then Events A and B are dependent. On the other hand, if the occurrence of Event A does not change the probability of Event B, then Events A and B are independent.

The probability of an event ranges from 0 to 1.

The sum of probabilities of all possible events equals 1.

Rule of Subtraction.

The probability that event A will occur is equal to 1 minus the probability that event A will <u>not</u> occur.

$$P(A) = 1 - P(A')$$

Suppose, for example, the probability that Bill will graduate from college is 0.80. What is the probability that Bill will not graduate from college? Based on the rule of subtraction, the probability that Bill will not graduate is 1.00 - 0.80 or 0.20.

Rule of Multiplication

The rule of multiplication applies to the situation when we want to know the probability of the intersection of two events; that is, we want to know the probability that two events (Event A and Event B) both occur.

Rule of Multiplication The probability that Events A and B both occur is equal to the probability that Event A occurs times the probability that Event B occurs, given that A has occurred.

$$P(A \cap B) = P(A)\ P(B|A)$$

Example

An urn contains 6 red marbles and 4 black marbles. Two marbles are drawn *without replacement* from the urn. What is the probability that both of the marbles are black?

Solution: Let A = the event that the first marble is black; and let B = the event that the second marble is black. We know the following:

- In the beginning, there are 10 marbles in the urn, 4 of which are black. Therefore, P(A) = 4/10.
- After the first selection, there are 9 marbles in the urn, 3 of which are black. Therefore, P(B|A) = 3/9.

Rule of Addition

The rule of addition applies to the following situation. We have two events, and we want to know the probability that either event occurs.

Rule of Addition The probability that Event A or Event B occurs is equal to the probability that Event A occurs plus the probability that Event B occurs minus the probability that both Events A and B occur.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note: Invoking the fact that $P(A \cap B) = P(A)P(B \mid A)$, the Addition Rule can also be expressed as:

$$P(A \cup B) = P(A) + P(B) - P(A)P(B \mid A)$$

Example

A student goes to the library. The probability that she checks out (a) a work of fiction is 0.40, (b) a work of non-fiction is 0.30, and (c) both fiction and non-fiction is 0.20. What is the probability that the student checks out a work of fiction, non-fiction, or both?

Solution: Let F = the event that the student checks out fiction; and let N = the event that the student checks out non-fiction. Then, based on the rule of addition:

$$P(F \cup N) = P(F) + P(N) - P(F \cap N)$$
$$P(F \cup N) = 0.40 + 0.30 - 0.20 = 0.50$$

Problem 1 :An urn contains 6 red marbles and 4 black marbles. Two marbles are drawn *with replacement* from the urn. What is the probability that both of the marbles are black?

(A) 0.16
(B) 0.32
(C) 0.36
(D) 0.40
(E) 0.60

Solution

The correct answer is A. Let A = the event that the first marble is black; and let B = the event that the second marble is black. We know the following:

- In the beginning, there are 10 marbles in the urn, 4 of which are black. Therefore, P(A) = 4/10.
- After the first selection, we replace the selected marble; so there are still 10 marbles in the urn, 4 of which are black. Therefore, P(B|A) = 4/10.

Therefore, based on the rule of multiplication:

$$P(A \cap B) = P(A)\ P(B|A)$$
$$P(A \cap B) = (4/10)*(4/10) = 16/100 = 0.16$$

References :

https://stattrek.com/probability/probability-rules.aspx

https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_BiostatisticsBasics/BS704_BiostatisticsBasics_print.html

https://byjus.com/maths/probability/